Series and Sequences Exercises

A. Determine whether each geometric series converges or diverges. If it converges, find its sum; If not, say why.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

2. $\displaystyle\sum_{k=0}^{\infty} \frac{2^{k+1}}{3^k}$

3. $\displaystyle\sum_{k=10}^{\infty} \frac{3^{k-1}}{2^k}$

4. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$

5. $\displaystyle\sum_{n=3}^{\infty} (-1)^n \frac{3}{2^n}$

6. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^2+n} + \frac{3^{2n}}{14^n} \right)$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$

B. True or False? Explain.

1. There exists an infinite geometric series with sum -4 and first term -1.

C. Determine whether each series converges or diverges.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{\sin^2(n)}{1+n^2}$

3. $\displaystyle\sum_{k=0}^{\infty} \frac{k}{e^k}$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

5. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+4n-3}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}+(-1)^n}{5^n}$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

9. $\displaystyle\sum_{n=1}^{\infty} (\arctan(n)+n)$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

11. $\displaystyle\sum_{n=2}^{\infty} \frac{(2n)!}{2^n n^2}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

13. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{\sin(100)}{\pi} \right)^n$

14. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$

D. Find the interval of convergence for the following power series.

1. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{4e} \right)^n$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n (n+1)}$

E. Taylor and Maclaurin Series.

1. Use Taylor series to evaluate $\displaystyle\int_0^1 x \cos(x)^3 \, dx$.

2. Find all antiderivatives of $\cos(x^2)$ using Maclaurin series.

3. Write a third degree Taylor Polynomial for $\sqrt{x}$ centered at $x = 4$.