Limits and Continuity Exercises

A. Are the following true or false? If true, explain why. If false, give a counter-example.

1. If $\lim_{x\to a} f(x)$ does not exist, then f is undefined at the point $x = a$.
2. If a function is not defined at $x = a$, then $\lim_{x\to a} f(x)$ does not exist.
3. If f and g are continuous on their domains which contain a, then $\lim_{x\to a} f(x) + g(x) = f(a) + g(a)$.
4. If $\lim_{x\to a} f(x)$ exists, then f is continuous at a.

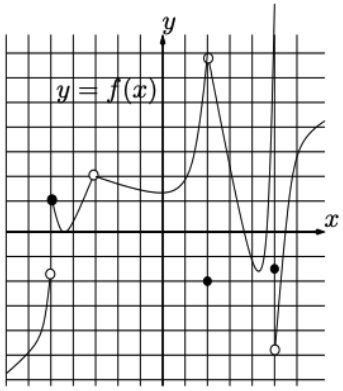
B. Evaluate the following limits (or say that the limit DNE):

1. $\lim_{x\to 3} \dfrac{x^2-9}{x+3}$
2. $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$
3. $\lim_{x\to \pi/2} \dfrac{\cot(x)}{\cos(x)}$
4. $\lim_{x\to 6} \dfrac{10}{x^2-36}$
5. $\lim_{x\to \infty} \tan(x)$
6. $\lim_{x\to \pi/2^+} \tan(x)$
7. $\lim_{x\to \infty} \dfrac{x^3+3x^2+4}{1-x^2}$
8. $\lim_{x\to \infty} \dfrac{\cos(x)}{x^2}$
9. $\lim_{x\to \infty} \dfrac{4x^4+3x^3}{7x^4+x}$
10. $\lim_{x\to \infty} \dfrac{10000x^3-x^2}{8x^4+2x+1}$
11. $\lim_{x\to 1^+} \dfrac{x^2+x+1}{x^2-1}$
12. $\lim_{x\to 0} \dfrac{(\cos^2(x)-1)(x+3)}{x}$
13. $\lim_{x\to 5} x^3 + e^x \sin(x)$
14. $\lim_{x\to 5} \dfrac{6\sin(x-5)}{x-5}$

C. For each function f, find a value of c so that f is continuous on $\mathbb{R}$:

1. $f(x) = \begin{cases} 2x & x \le c \\ x^2+1 & x > c. \end{cases}$
2. $f(x) = \begin{cases} 2x+c & x < 2 \\ x^2+cx+1 & x \ge 2. \end{cases}$

D. Answer the following questions based on the graph (each box has width 1).



1. At what points a does $\lim_{x\to a} f(x) = L$ but $L \neq f(a)$?
2. At which points is f not continuous?
3. Does $\lim_{x\to 2^-} f(x)$ exist? If it does, what is its value?
4. Does $\lim_{x\to 2^+} f(x)$ exist? If it does, what is its value?
5. Does $\lim_{x\to 2} f(x)$ exist? If it does, what is its value?
6. What is $f(2)$?

E. Answer the following questions based on the function f defined below.

$$f(t) = \begin{cases} 1+t & t < 0 \\ t^2+1 & 0 \le t < 1 \\ 3 & t = 1 \\ t+4 & t > 1 \end{cases}$$

1. What is $\lim_{t\to 0} f(t)$?
2. What is $\lim_{t\to 0^+} f(t)$?
3. What is $\lim_{t\to 0^-} f(t)$?
4. Where is f continuous?

Answers (in no particular order)

- -5, -3, 2, 5
- 0
- 6
- -1
- False ($f(x) = \frac{x^2}{x}$ is not defined at 0, but $\lim_{x\to 0} f(x) = 0$)
- 1
- 6
- 1
- -3, 2
- DNE
- False ($f(x) = \frac{|x|}{x}$ is not continuous at 0, but $\lim_{x\to 0} f(x) = 0$)
- $-\infty$
- 1
- $125 + e^5 \sin(5)$
- DNE
- ∞
- -2
- True (Since f and g are continuous, so is $f+g$. Then by the def. of continuity, $\lim\limits_{x\to a} f(x)+g(x) = f(a)+g(a)$)
- 0
- $(\infty, 1) \cup (1, \infty)$
- False (If $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$ then $\lim\limits_{x\to a} f(x)$ but $f(0) = 1$)
- 1
- yes, 6.8
- 0
- yes, 6.8
- 1
- yes, 6.8
- ∞
- $\frac{4}{7}$