Limits and Continuity Exercises

A. True or false? If true, explain why. If false, give a counter-example.

1. If $\lim_{x\to a} f(x)$ does not exist, then f is undefined at the point $x = a$.
2. If a function is not defined at $x = a$, then $\lim_{x\to a} f(x)$ does not exist.
3. If f and g are continuous on their domains which contain a, then $\lim_{x\to a} f(x) + g(x) = f(a) + g(a)$.
4. If $\lim_{x\to a} f(x)$ exists, then f is continuous at a.

B. Evaluate the following limits using the limit laws. Write which law you are using in each step.

1. $\lim_{x\to 4} x^2 + 3x - 1$
2. $\lim_{x\to -2} \sqrt[3]{x^4+1}$
3. $\lim_{x\to 0} \left(\sqrt{x}+1\right)^{100}$
4. $\lim_{x\to 2} \dfrac{x-3}{x+4}$

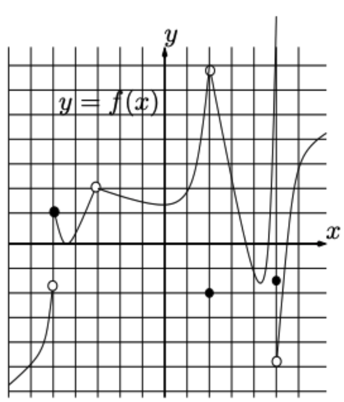
C. Evaluate the following limits (or say that the limit DNE):

1. $\lim_{x\to 3} \dfrac{x^2-9}{x+3}$
2. $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$
3. $\lim_{x\to \pi/2} \dfrac{\cot(x)}{\cos(x)}$
4. $\lim_{x\to 6} \dfrac{10}{x^2-36}$
5. $\lim_{x\to \infty} \tan(x)$
6. $\lim_{x\to \pi/2^+} \tan(x)$
7. $\lim_{x\to \infty} \arctan(x)$
8. $\lim_{x\to \infty} \dfrac{x^3+3x^2+4}{1-x^2}$
9. $\lim_{x\to \infty} \dfrac{\cos(x)}{x^2}$
10. $\lim_{x\to \infty} \dfrac{4x^4+3x^3}{7x^4+x}$
11. $\lim_{x\to \infty} \dfrac{10000x^3-x^2}{8x^4+2x+1}$
12. $\lim_{x\to 1^+} \dfrac{x^2+x+1}{x^2-1}$
13. $\lim_{x\to 0} \dfrac{(\cos^2(x)-1)(x+3)}{x}$
14. $\lim_{x\to 5} x^3 + e^x \sin(x)$
15. $\lim_{x\to 5} \dfrac{6\sin(x-5)}{x-5}$
16. $\lim_{x\to 0} \dfrac{\sin(x^2)}{x}$
17. $\lim_{x\to \infty} \dfrac{\sqrt{3x^4-x^2}+x^2}{4x^2+1}$
18. $\lim_{x\to 0^+} e^{-1/\ln(x)}$
19. $\lim_{x\to \infty} \left(1+\dfrac{4}{3x}\right)^x$
20. $\lim_{x\to 0} x^2\cos\left(\dfrac{1}{x}\right)$

D. For each function f, find a value of c so that f is continuous on $\mathbb{R}$.

1. $f(x) = \begin{cases} 2x & x \le c \\ x^2+1 & x > c. \end{cases}$
2. $f(x) = \begin{cases} 2x+c & x < 2 \\ x^2+cx+1 & x \ge 2. \end{cases}$

E. Answer the following questions based on the graph (each box has width 1).



1. At what points a does $\lim_{x\to a} f(x) = L$ but $L \neq f(a)$?
2. At which points is f continuous?
3. At which points is f not continuous?
4. Does $\lim_{x\to 2^-} f(x)$ exist? If it does, what is its value?
5. Does $\lim_{x\to 2^+} f(x)$ exist? If it does, what is its value?
6. Does $\lim_{x\to 2} f(x)$ exist? If it does, what is its value?
7. What is $f(2)$?

F. Answer the following questions based on the function f defined below.

$$f(t) = \begin{cases} 1+t & t<0 \\ t^2+1 & 0 \le t < 1 \\ 3 & t = 1 \\ t+4 & t > 1 \end{cases}$$

1. What is $\lim_{t\to 0} f(t)$?
2. What is $\lim_{t\to 0^+} f(t)$?
3. What is $\lim_{t\to 0^-} f(t)$?
4. Where is f continuous?

G. Describe the points of discontinuity of the following functions.

1. $f(x) = \begin{cases} \dfrac{\sin(3x^2)}{x-1} & 0 \le x \\ \dfrac{(x^2+1)(x-2)}{x^2-4} & x < 0 \end{cases}$

2. $f(x) = \begin{cases} \dfrac{x-5}{x^2-7x+10} & 2 < x \\ \dfrac{1}{1-2^{(x-2)}} & -1 \le x < 2 \\ \ln(-x) & x < -1 \end{cases}$

H. Use the IVT to show that each equation has a solution on the given interval.

1. $\tan(\cos(x)) = \frac{1}{x^2+1}$, $[0, 2]$
2. $x^2 = e^x + 4$, $[-3, 0]$
3. $\ln\left(x^2-1\right) = \csc(x)$, $[6, 8]$