Derivatives Exercises

A. Are the following true or false? If true, explain why. If false, give a counter-example.

1. If a function is continuous at a, then $f'(a)$ exists.
2. If a function is differentiable, then its derivative is differentiable.

B. For each f, find f' using the limit definition of the derivative.

1. $f(x) = 3$
2. $f(x) = \sqrt{x+4}$
3. $f(x) = 2x^2 + 3x$
4. $f(x) = \sin(x)$

C. Which functions' derivative is given by the following limits?

1. $\lim_{h \to 0} \frac{\tan(x+h) - \tan(x)}{h}$
2. $\lim_{h \to 0} \frac{\sqrt{2x+2h-3} - \sqrt{2x-3}}{h}$
3. $\lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 + 1}{h}$

D. Differentiate each of the following.

1. $4x^{3/4}$
2. $\cot(\sin(x))$
3. $x^{\ln(2x)}$
4. $\ln \frac{(x^2-4x+1)^3}{(3x+5x^2)^8}$
5. $\sin^{-1}(\pi x)$
6. $e^{\frac{1}{\cos(\sqrt{x})}}$
7. $\csc(4x^2)$
8. $\sqrt{1+x^2}$
9. $\cos(\ln(\sin(x)))$
10. $\arctan(\pi + \ln(x))$
11. $\left(\frac{1}{x}\right)^x + \left(\frac{1}{x}\right)^2 + \left(\frac{1}{2}\right)^x$
12. $(x^3 + 2x^5)(x^9 - 5x^7 + 3)$

E. For each f, find f' and f'' using any method you want.

1. $f(x) = 3x^3 - \frac{4}{x^2}$
2. $f(x) = x\sin(x)$
3. $f(x) = \frac{x^2+3}{x-4}$
4. $f(x) = x^2\cos(x) + x\tan(x)$
5. $f(x) = \sin(x)\cos(x)e^x$
6. $f(x) = x^5 + x^4 + \pi^3 + x^2 + x + 1$

F. Suppose you know the following information: $f(2) = -3$, $f'(2) = 4$, $g(2) = 2$, $g'(2) = 3$, $h(2) = -2$, and $h'(2) = -4$. Evaluate the derivative of each of the following at $x = 2$.

1. $3f(x) - 6h(x)$
2. $(f(x) + g(x))^4$
3. $f(g(x))$
4. $(g(x))^{h(x)}$
5. $e^{h(x)}$

G. Find the 100th derivative of each function.

1. $f(x) = x^{70}$
2. $f(x) = xe^x$
3. $f(x) = -\cos(x)$
4. $f(x) = 2^{-x}$

H. Find the tangent line to the following curves at the given point.

1. $y^3 x = 3x + 4y$, $(0,0)$
2. $\tan(x+y) = \tan(xy)$, $(0,0)$
3. $\frac{(x-2)^2}{9} + \frac{(y-5)^2}{4} = 1$, $(2,7)$
4. $y^x = 4x$, $(4,2)$
5. $y^4 - 4y^2 = x^4 - 9x^2$, $(3,-2)$
6. $\sin\left(\frac{\cos(y)}{\pi}\right) = 3xy$, $(0, \pi/2)$

I. Where is the tangent line to the curve defined by $xy^2 = x^2 + 2y$...

1. parallel to the x-axis?
2. vertical?
3. perpendicular to $3y = 5x + 1$ and x and y are both integers less than 3 in absolute value?

Answers (in no particular order)

- $f'(x) = \frac{1}{2}(x+4)^{-1/2}$
- $f(x) = 3x^2 - 1$
- $y = -\frac{3}{4}x$
- 36
- $x^{\ln(2x)-1}(\ln(x) + \ln(2x))$
- $f'(x) = 4x + 3$
- $\frac{1}{2}(1+x^2)^{-1/2}$
- $f'(x) = 0$
- False ($f(x) = \frac{|x|^3}{2x}$ is differentiable and has derivative $f'(x) = |x|$, which is not differentiable)
- $f'(x) = \cos(x)$
- $f(x) = \tan(x)$
- $y = 7$
- $\frac{-4}{e^2}$
- $f^{(100)} = \ln(2)^{100}2^{-x}$
- $-\csc(4x^2)\cot(4x^2)8x$
- $-\csc^2(\sin x)\cos(x)$
- $f'(x) = 2x\cos(x) - x^2\sin(x) + \tan(x) + x\sec^2(x)$,
  $f''(x) = \left(2 - x^2\right)\cos(x) - 4x\sin(x) + \sec^2(x)\left(2 + 2x\tan(x)\right)$
- -28
- $y = \frac{\pi}{2} - \frac{3\pi^2}{2}x$
- $f^{(100)=} - \cos(x)$
- $y = -x$
- $3x^{-1/4}$
- $(3x^2 + 10x^9)(x^9 - 5x^7 + 3) + (x^3 + 2x^5)(9x^8 - 35x^6)$
- $f'(x) = \sin(x) + x\cos(x),\ f''(x) = \cos(x) + \cos(x) - x\sin(x)$
- $e^{\frac{1}{\cos(\sqrt{x})}}\cos(\sqrt{x})^{-2}\sin(\sqrt{x})\frac{1}{2}x^{-1/2}$
- $-\ln(3) - \frac{3}{4}$
- $-\sin(\ln(\sin(x)))\frac{1}{\sin(x)}\cos(x)$
- $f^{(100)}(x) = 0$
- $(2, -1)$
- $y = -2 - \frac{27}{8}(x-3)$
- $f(x) = \sqrt{2x-3}$
- False ($f(x) = |x|$ is continuous at 0, but $f'(0)$ DNE)

- $f'(x) = e^x(\cos^2(x) + \sin(x)\cos(x) - \sin^2(x))$,
  $f''(x) = f'(x) + e^x\left(-4\cos(x)\sin(x) + \cos^2(x) - \sin^2(x)\right)$
- 12
- $y = 2 + \left(\frac{1}{8} + \ln(1/\sqrt{x})\right)(x-4)$
- $(0,0)$, $(2,2)$
- $\frac{(3x+5x^2)^8}{(x^2-4x+1)^3} \cdot \frac{3(x^2-4x+1)^2(2x-4)(3x+5x^2)^8 - 8(x^2-4x+1)^3(3x+5x^2)(3+10x)}{(3x+5x^2)^{16}}$
- $f'(x) = 5x^4 + 4x^3 + 2x + 1$, $f''(x) = 20x^3 + 12x^2 + 2$
- $\left(\frac{1}{x}\right)^x\left(\ln\left(\frac{1}{x}\right)\right) - 2x^{-3} + \ln\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^x$
- $f'(x) = 9x^2 + 8x^{-3}$, $f''(x) = 18x - 24x^{-4}$
- $f^{(100)}(x) = 100e^x + xe^x$
- $(-1,-1)$
- $\frac{\pi}{\sqrt{1-(\pi x)^2}}$
- $f'(x) = \frac{(2x)(x-4)-(x^2+3)}{(x-4)^2}$, $f''(x) = \frac{(2x-8)(x^2-8x+16)-(x^2-8x-3)(2x-8)}{(x^2-8x+16)^2}$