Derivative Application Exercises

A. For each of the following functions, find (a) the intervals on which they are increasing/decreasing, (b) the intervals on which they are concave up/down, (c) the coordinates of any local maxima or minima, (d) points of inflection. Then sketch the graph.

1. $f(x) = 2x^2 + 3x$
2. $f(x) = x^3 + 17x^2 + 2$
3. $f(x) = \sin(x) - \cos(x)$ on $[-2\pi, 2\pi]$
4. $f(x) = e^{\cos(x)}$

B. Estimate each of the following.

1. $\sqrt{9.1}$
2. $\sqrt[5]{31.9}$
3. $\sin(\pi + 0.1)$
4. $\ln(e - 0.1)$
5. $e^{0.1}$

C. Kinematics

1. What is the difference between speed and velocity?
2. If the displacement from the origin of a particle after time t is given by $d(t) = 3\cos^2(t) + 4^t$, write the formula for the function of the particles velocity and acceleration.
3. A ball is thrown up into the air. Its height after t seconds is given by $h(t) = -2t^2 + 10t$. (a) how high is the ball after 2 seconds? (b) What is its average velocity during the first two seconds? (c) How fast is the bullet traveling after 2 seconds? (d) What is the acceleration of the bullet at 2 seconds?

D. Related Rates

1. For all positive values of c, the parabola $f(x) = x^2 + 2x - c$ has one positive root. Find the rate at which this root is changing with respect to the rate at which c is changing.
2. Two ships start at the same port depart at the same time. One travels North at a rate of 30 m/s and the other travels East at a rate of 40 m/s. How fast is the distance between the two ships changing after 10 minutes?
3. I have a cylindrical vat of water 3ft in diameter, and a hose that fills it up at a rate of 4 ft^3/min. It also leaks $\frac{1}{2}$ ft^3 every minute. If the vat is initially empty, how fast is the height of the water changing after 10 minutes?
4. When I look at a tree in the distance, it looks small, but as I walk closer, the tree appears to be growing. I start 100 meters away from the tree and hold my arm (0.5 meters long) out with ruler in my hand and the tree reaches the 10 cm mark. I then start walking towards the tree at 1.5 m/s. How fast is the observed height of the tree changing after 10 seconds? How about when I am 50 m from the tree?
5. Sand is falling out of a pipe at a rate of 4 m^3/s, forming a conical pile. Due to the crystalline structure of the sand, the radius of the pile is always twice the height. How fast is the height of the sand pile changing when the height is 10 m?
6. A 10 ft ladder is leaning against a wall but the ground is slippery and it starts to slide. After time t, the bottom end of the ladder is $x(t) = 2t^2 + 5$ ft away from the wall. How fast is the height of the top of the ladder changing when the bottom of the ladder is 7 feet from the wall? How fast is the area under the ladder changing at the same time?
7. A hot air balloon is taking off 80 feet away from me and is rising at 7 m/s. As it does, I adjust my head's angle to keep looking at it. How fast is this angle changing when the balloon is 70 meters high?

E. Optimization

1. A rectangle has its base on the x-axis and its top two points lie on the parabola given by $f(x) = 16 - x^2$. What is the maximum area of this rectangle?

2. A movie company is trying to decide how many explosions they should put in their movie to maximize profits. The producers will allow anywhere between 0 and 100. The cost for x explosions is $10 + \sqrt{x}$ thousand dollars. Market research suggests that x explosions will bring in $12 + 10\arctan(\sqrt{x})$ thousand dollars at the box office. How many explosions should the movie company use to maximize profits?

3. We want to construct a box with no lid to maximize volume. What is the most volumetric box I can obtain with only 10 ft^2 of material if I insist the base of the box is a square?

4. I want to make a cylindrical metal can that holds 1 Liter (1000 cm^3) of water. What are the proportions of the can that will minimize the amount of metal I need to use?