Complex Numbers Exercises

1. For an arbitrary $z = x + iy$, when is $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ and when is it not?

2. Two complex numbers z and w are given by $z = 3 + 4i$ and $w = 2 - i$. Plot (a) $z + w$, (b) $w - z$, (c) wz, (d) z/w, (e) $z^*w + w^*z$, and (f) w^2.)

3. Find the polar representation of the following complex numbers:

   (a) $-\sqrt{3} + i$

   (b) $2 - 2i$

   (c) $-i3/2$

   (d) $(-1 - \sqrt{3}i)/2$

   (e) i

   (f) -2

4. Does $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$ hold for $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$? Check that with $z_1 = \sqrt{3} + i$ and $z_2 = (3/2)(1 + i\sqrt{3})$.

5. By considering the real and imaginary parts of the product $e^{iA}e^{iB}$ prove the standard formulas for $\cos(A + B)$ and $\sin(A + B)$.